

III Nordic GeoGebra Conference, Tartu Estonia, sept. 14th-16th

Abstracts of plenary talks, workshops and talks

Plenary talks

Title: An overview, what happened in the GeoGebra Community in the last year.

Speaker: Balazs Koren from the GeoGebra Team.

Abstract: Software: In 2011 version 4 was released. GeoGebraTube started in September 2011 and we reached 5000 uploaded materials in March 2012.

After the release of version 4, the development of version 4.2 and 5.0 started. Great new features were included in the ready-to-test beta versions, as CAS in version 4.2 and real 3D in 5.0.

Community: In 2011 we had 65 Institutes, in May we are to celebrate the establishment of the 100th GeoGebra Institute. Over 6 Million people downloaded the software in 2011.

Huge numbers, a fast growing community. What is your role in this big family? How can the community help you, how can you help the community?

Title: GeoGebra as a Tool for developing Mathematical Competence.

Speaker: Svein Torkildsen, Norwegian Centre for Mathematics Education, Norway.

Abstract: Since the Early Nineties I have been inspired by the Ideas of Realistic Math Education developed by The Freudenthal Institute, University of Utrecht. I have been looking for real problems as a starting point for my students to make them problem solvers and give opportunities to develop Conceptual Understanding. The challenge for the students has been to make models so they could explore the problems. For me as a teacher the challenge has been to put attention on the mathematics. Most of the math teachers in Norway created ICT-courses for their students, focusing on learning to know the tools. I have tried to integrate the tools in the ordinary math lessons, both as tools for the teacher in visualizing math and as tools for students. Sometimes the students had to work on files prepared for a particular purpose. Other times they had to develop the files from scratch. In this talk I will give examples from all this areas.

Title: GeoGebra as a tool to support thoughtful discussion and open investigations in mathematics

Speakers: Ingólfur Gíslason, University of Iceland and Valgarð Jakobsson, Framhaldsskólinn í Mosfellsbæ, Iceland.

Abstract: We explore the use of GeoGebra as a supporting tool for a pedagogy that emphasises thoughtful discussion in the classroom and open investigations. A first mathematics course in our upper-secondary school is an introduction to functions and graphs. GeoGebra is used in several ways by the pupils. For example as a simple function graphing tool, as a tool for modelling mathematical and practical situations, and as tool for representing co-variation of geometric points. We discuss some of these uses and what mathematics the pupils have the opportunities to learn from them. We present positive learning experiences we have observed and how the possibilities of the software have forced us to rethink mathematics teaching and learning.

Title: The use of pre-programmed applets in mathematics lessons in years 2 and 7 in Danish Schools.

Speakers: Rikke Teglskov, University College Lillebælt and Bo Kristensen, Højby Skole, Odense, Denmark.

Abstract: In this plenum talk, we will focus on two ways of using applets

1. Using applets as a practicing tool
2. Using applets as a way of didactically designing lessons

Pre-programmed applets can be used with the purpose of practicing certain mathematical skills. The applet creates a task for the student to solve, and no two students get the same task. Whenever a task is solved a new one can easily be launched by refreshing the applet. This makes it possible to design applets with emphasis on practising a specific skill and in the meantime the students will practice some of their skills in using GeoGebra as a tool.

The programming of applets in GeoGebra instead of using the full version of the program allows for the math teacher to compose and micro-manage lessons with emphasis on investigations of specific parts of the curriculum.

Based on examples and experiences from 2nd and 7th graders (8 and 14 years of age) we teach we will discuss the subjects below.

- a) Pros and cons of using applets instead of the full version of GeoGebra.
- b) Collaborative learning and students verbalizing mathematics.
- c) Learning by discovery - students as mathematical explorers and re-inventors oppose to re-producers.
- d) Flipping the classroom - A step away from the teacher as the oracle. The students become the ones defining mathematics.
- e) Students' mathematical ownership.

Title: Assessment for Learning - when using GeoGebra

Speaker: Malin Christersson, Katedralskolan Lund, Sweden.

Abstract: As we introduce technological artifacts as tools for learning, we must also change our methods of assessing mathematical thinking. Software such as GeoGebra enables us to spend less time on repetitive exercises and more time on doing investigations and modeling. There is an abundance of rich mathematical problems available, but how do we rephrase them in order to encourage creativity using technology? In order to do so, we must not only rethink the questions given to the students, but also rethink how we assess the students' processes and provide feedback. Furthermore, how do we encourage all teachers to make this transition of teaching?

Title: Curriculum reform – a new direction for math education in Finland?

Speaker: Leo Pahkin, The Finnish National Board of Education, Finland.

Abstract: The modern society is in a rapid change. The demand for skills has changed from routine manual, nonroutine manual and routine cognitive tasks towards nonroutine analytic and nonroutine interactive tasks. This means that mathematics has a bigger and more important role in developing such skills. How the Finns are going to respond for this new challenge?

Workshops

Title: Teaching statistics using GeoGebra – examples from a secondary school in Denmark

By: Frede Dybkjaer, Danish GeoGebra Institute, Denmark.

Abstract: The curriculum mentions mean value, standard deviation, quartile set, boxplot, histogram, the normal distribution and X²-test. The workshop will include several examples from my teaching, where students have measured heights and weights, rolled dices (including a loaded dice), measured distances in a gameplay, used a pack of cards, etc.

Title: Using GeoGebra in the earliest school years.

By: Rikke Teglskov, University College Lillebælt and Bo Kristensen, Højby Skole, Odense, Denmark.

Abstract: How can you introduce GeoGebra to very young students and why do it at all? What benefits are there for age group? Which mathematical topics could be used for this purpose? In this workshop we will give some ideas for using GeoGebra in your mathematics lessons with even the youngest students in school.

Based on the thoughts of didactical design we will investigate the possibilities of adapting menu bars and exporting Geogebra files as html files for this age group in order for them to investigate both mathematical topics as well as developing their GeoGebra skills.

We will go through the learning potentials of incorporating GeoGebra in different mathematical topics. The workshop will be based on a website including preprogrammed applets and GeoGebra files.

Title: How to design simple applets in GeoGebra using the randombetween function

By: Rikke Teglskov, University College Lillebælt and Bo Kristensen, Højby Skole, Odense, Denmark.

Abstract: The possibilities and purposes of using GeoGebra are numerous and the creation of applets adds another dimension to math in school. By designing applets you can help the students discover specific didactical points, practice mathematical skills or practice using different GeoGebra skills. In this workshop we will explore the possibilities of applets and in specific the randombetween function and how it can be used in various ways to make GeoGebra compose different challenges for the students.

We will present ready to use applets and step by step we will go through how some of them are made. We expect the participants in this workshop to be familiar with the basic functionalities of GeoGebra.

Title: Designing curves, from parabolas to Bezier-curves

By: Christoph Kirfel, University of Bergen, Norway.

Abstract: Designing nice curves is desired when drawing pictures or designing clothes or tools. Most drawing programs on a computer have a drawing device. What kind of mathematics is hidden in these devices. Can we design our own drawing device with GeoGebra?

The pupils (17 years old) are familiar with drawing programs on their computers. They are also familiar with parabolas from their mathematics classes. On our way to develop our own drawing tool we therefore first introduce curves consisting of consecutive pieces of parabolas. We discuss the concept of smoothness and work out the conditions for smooth transitions from piece to piece. We also discuss the

disadvantages of the model when used as a drawing device. In order to overcome the difficulties mentioned we look at a broader type of curves, the so-called Bezier-curves. These have a number of nice properties which are appropriate for our goals. In the end we build a new drawing device by Bezier-curves and show how to fit Bezier-curves to various tasks.

We also present the results from the classroom presentation and discuss obstacles and advantages with the approach.

Requirements: The participants of the workshop need to have their own PC where they can work with GeoGebra.

Title: GeoGebra and other free web based resources in elementary school

By: Ulrika Ryan, Lunds kommun, Sweden.

Abstract: In this workshop you will try some GeoGebra ideas that let the youngest pupils experience patterns and coordinates in basic math. You will also get an overview of other free web based resources that can be useful when teaching math in elementary school.

Title: Mathematical modelling with GeoGebra

By: Minna Jokela, Laura Kauppinen and Jussi Kytömäki, Ylöjärvi Upper Secondary School, Finland.

Abstract: The workshop uses GeoGebra to explore mathematical models related to physical phenomena.

Examples: Wave motion, Forces, Review of progressive motion in (t,a) , (t,v) and (t,s) co-ordinate systems.

Title: What is mathematics all about? Inquiry, conjectures and proofs with GeoGebra

By: Sigbjørn Hals, Måløy vidaregåande skule, Norway

Abstract: In this workshop we will look at some nice mathematical problems, conjectures and proofs, which my Norwegian students aged 16 - 18 have worked with. I will start with a short introduction on how to stimulate mathematical thinking and problem solving. The participants will then work with stimulating challenges, and get some useful advice on how to make the students able to find their way through Pólyas four steps in problem solving.

Title: Using GeoGebra together with Google sites as platform for evaluation and teaching of mathematics

By: Martin Thun Klausen and Troels Christensen, Denmark.

Abstract: How can we use GeoGebra in conjunction with Google Sites to evaluate our students mathematical skills or to teach our students the necessary skills?

Based on a specific mathematical topic it gradually will be explained, how it can be made into a Google site that contains text, tasks, applets of the subject. The participants can then work on a chosen topic, or start with our examples.

In addition we look at tasks from the written test in mathematics in the ninth grade in Denmark, which can be solved in GeoGebra.

Title: Art in GeoGebra

By: Jonas Hall, Sweden

Abstract: Viewing the graphics view as a canvas, it is possible to make beautiful patterns using mathematics. Participants will learn how to paint the canvas "by hand", but more importantly, how to use mathematical functions of two variables to colour the canvas. Towards the end of the workshops we will see how to export the graphics view as an image and how to automate this using scripts. Finally we will try a Python program to produce high definition images like those shown at the conference

Concepts we will address: Tracing, dynamic colors, sliders, animation, functions of two variables, rigid polygons, using the spreadsheet, input boxes, command buttons, scripting in GeoGebra and Python.

Although all of this will be explained in the workshop, participants will benefit from having previous experience with at least some of these concepts, (alternatively being a fast learner).

Title: Pupils creating their own tasks for Geogebra with storylines

By: Nanna Filt Petersen and Heidi Berggren Brøndal Pedersen, Brøndbyvester Skole, Denmark.

Abstract: Mathematics with engaged students who take ownership and responsibility. Mathematics with a high degree of creativity and familiarization. Mathematics with diversity and unlimited potentials for differentiation.

Our teaching sequences in two Danish grades 7th (13 – 14 years old) were inspired by Dewey's 'learning by doing' – if we want the students to learn how to solve tasks in mathematics, then why shouldn't they try to create tasks on their own too? The level of metacognitive thinking was high according to our experiences with the young students who engaged in choice of topic and story for their task, which was supposed to be solved by the use of GeoGebra.

The students work on questions like: What does it mean to be a good task in mathematics? Or to be a good solution? Or even: what should a good guide for the teacher include? gave results, not only in the form of fine and useful GeoGebra tasks but also new insight to the students about the goals and meaning with problem solving in mathematics.

Examples of the students' GeoGebra tasks will be displayed in our workshop. We will present the projects and the ideas behind, in particular the advantage of taking the students ideas and creativity as the starting point. Workshop participants will play the role of the students in the project, and we support the production of a task based on a topic and a story – a GeoGebra task, of course!

Title: Some ideas of composing effective GeoGebra worksheets

By: Jane Albre-Andersen, Norway/Estonia.

Abstract: While composing electronic teaching material raises often up question how to design it in the way that it is easy for students to use and it helps to acquire the essential information. In the workshop some principles for creating effectively designed materials will be given. Workshop is based on practical examples that illustrate these principles. Participants are expected to be active in improving these electronically given examples. Additionally, GeoGebra tools that help to implement given designing principles will be revised. Examples are chosen from different school-levels and from various topics in mathematics to present some ideas how to illustrate mathematics teaching.

Talks

Title: Reifying instrumental genesis - What is a triangle?

By: Hannu Korhonen, GeoGebra Institute of Finland.

Abstract: The process of learning to use a tool has two directions: one from the user to the tool (learning to use) and the other from the tool to the user (conceptions formed). Consequently, every time you use your new tool, you are becoming more and more agile and versatile. On the other hand, and what is more important in learning, the tool is modifying your concepts and ways of thinking. In the beginning, the change is slow and gradual, but accelerating with time and experience. The theoretical concept behind all of this is sometimes called instrumental genesis.

But that is not a theory only. Every experienced Geogebra user knows the phenomenon. Along with time and experience your concepts are becoming more dynamic and you will find that you are thinking more eloquently and versatilely. This is happening also with as simple and basic concepts as a triangle. In my talk, I am considering different ways of drawing a triangle, the attributes of a triangle formed, and the consequences for teaching and learning mathematics in schools.

Title: The Euler number e

By: Torger Johannes Nilsen, NGI Norway

Abstract:

Plato said, "God is a geometer". Jacobi changed this to, "God is an arithmetician." Then came Kronecker and fashioned the memorable expression, "God created the natural number, and all the rest is the work of man".

Felix Klein

I wonder if all these mathematicians have forgotten the Euler number. In the famous book Complex analysis by Lars V. Ahlfors, Ahlfors defines the exponential function as the solution of the differential equation $f'(x) = f(x)$ with the initial value $f(0) = 1$.

I want to present a lecture I have done where we use GeoGebra and the above definition to rediscover the Euler number. We will also meet the Euler number by working with the integral of the function

$g(x) = \frac{1}{x}$ in GeoGebra.

The students do not need any prior knowledge about differential equations or integral calculus to benefit from the lecture.

Title: GeoGebra in vocational training

By: Stig Eriksen, Dahlske vgs, Norway.

Abstract: In Norway, the subjects within vocational programs can be divided into two parts. The students meet their future profession in some of their subjects and they meet theoretical demands in some subjects that are common for all the vocational programs. It is often assumed that the theory of the common subjects can cause students to drop out of school. Mathematics is one of these common subjects. During my presentation I will, for a few different vocational programs, show examples on how GeoGebra can be used to lessen the gap between the vocational subjects and the common subjects.

Title: GeoGebra in an Icelandic classroom and some advanced uses.

By: Kristján Einarsson, Framhaldsskólinn í Mosfellsbæ, Iceland

Abstract: Kristján is a young teacher with one year's teaching experience in an Icelandic upper secondary school. Here he will talk about his experience of using GeoGebra, his results in a study on the use of GeoGebra in Icelandic classrooms and give examples of his own use.

In the study, teachers who knew GeoGebra were asked how they used the program and how often. The results give us the idea of how the program is used in practice and how we could classify GeoGebra according to their different methods and purposes.

The examples from teaching mainly shows work from teaching basic calculus, i.e. lines and parabolas. Some of those problems use a premade GeoGebra file with some advanced uses of the programming options.

Title: Lecture on teaching high-school level calculus, definition of derivative, etc.

By: Vilhjálmur Þór Sigurjónsson, Menntaskólinn í Kópavogi, Iceland

Abstract: The module Stæ403 (Math403) included three groups of students taught by three teachers. At the beginning of the semester the decision was taken to adopt an unconventional approach, focusing very much on cooperation between teacher and students and moving responsibility onto the students with the aid of the latest technology such as Geogebra and WolframAlpha, Kahn-academy, etc.

At the beginning of the course, students had to install the program Geogebra and received a brief instruction on how it worked. Subsequently there were projects that students could choose to solve using Geogebra.

Teachers made considerable use of a digital tablet and Geogebra to draw graphs and provide precise explanations of calculations.

This presentation will discuss a survey presented to students at the end of the course. It will also include a discussion between the teachers involved on how this approach differs from conventional teaching methods.

Title: Circle and regular polygons with GeoGebra. Worksheets

By: Merlin Saulep, Riina Taidre and Eno Tõnisson, Tartu Ülikool, Estonia

Abstract: A new curriculum of mathematics in Estonia pays more attention to the computers: a student must be able to construct geometric shapes by hand and also using the computer. This inspired to compose worksheets on a Bachelor's thesis and construct worksheets on Thales' theorem, tangent line and secant line, a triangle inscribed circle, a triangle circumscribed circle, regular and irregular polygons. The other task was to examine teachers opinion on these worksheets and suitability of GeoGebra for mathematics lessons. There was also measured how students test results changed, comparing with students who learned this topic without worksheets and GeoGebra. Finally was asked from students how easily they find working with GeoGebra and worksheets.

Title: GeoGebra.tv - The future of GeoGebra tutorial videos

By: Janne Cederberg, Opetus.tv, Finland.

Abstract: Sure you've seen some brilliant GeoGebra worksheets so far, for example by Daniel Mentrard and many others. Upon seeing what was accomplished, the feeling of awe and amazement permeated your whole being...and then it dawned on you: "I wish I knew how to create something like that!" You then set out to examine the worksheet (if it was accessible) in hopes of discovering how it was created...only to realize you were either not able to understand it or didn't have the time to figure it out. Inspired, yet slightly disappointed, you went on with your day...

Imagine if the author had explained the creation process of the worksheet on video and you could follow along while simultaneously trying things out in GeoGebra! And imagine students being drawn to learning GeoGebra through tutorial videos and in the process learning math, physics, etc. (compare with Conrad Wolfram's "Teaching kids real math with computers" presentation @ TED: http://www.ted.com/talks/lang/en/conrad_wolfram_teaching_kids_real_math_with_computers.html)

What do you think GeoGebra.tv should be? Are you interested in getting involved in building the future?

Title: Start using Geogebra in Lithuania: success and failure

By: Vilija Šileikienė, Klaipėda Ažuolyno gymnasium and Laura Stepanauskienė Vilnius Balsiu Basic school, Lithuania.

Abstract: We first met Geogebra in 2011 autumn conference in Vilnius. In conference we discovered that the program has great possibilities to use it in math classes. We came home with desire to learn the program and to show it to our students.

Currently we use Geogebra in math classes with 10 and 12 grade students. We designate students do homework or project works with Geogebra. Our report will present more what is a way of using Geogebra:

- Drawing function graph;
- Function transformations;
- Solving equations and systems of equations graphically;
- The tangent line for a function;
- Finding the maximum and minimum values of function;
- Students' short term project "reconstruction of the school yard";

We will share our first experiences of start using Geogebra. What are we happy about and what are the problems we face. Report is intended for beginners to start work with Geogebra.

Title: GeoGebra in the 10th grade mathematics

By: Ričardas Kudžma, Ala Rožkova and Virginija Vilkotyė, Vilnius University, Lithuania.

Abstract: Now it is difficult to imagine teaching mathematics without using IT technologies. One of the reasons not to do that is a big age of mathematics teachers. They are not familiar with the new technologies. Mathematics programs do not require obligatory using IT at mathematics classes. The first author of this presentation was also not very active in using IT. But after GeoGebra conference in Reykjavik he changed his mind. He introduced GeoGebra for the third year prospective mathematics

teachers' students at mathematics didactics classes in 2010 autumn. This spring two of them, Ala Rožkova and Virginija Vilkotyte had three months pedagogical practice at school. They were asked to apply GeoGebra during teaching mathematics at the 10th grade. They were very active and exceeded their first tasks many times. They created the first Lithuanian GeoGebra blog: www.matematika-2012.blogspot.com

We think that GeoGebra is a good software.

Title: ICT-supported Creative reasoning for Problem solving

By: Carina Granberg, Umeå University, Sweden

Abstract: One of the problems in mathematics education is identified as that classroom activities often focus on routine-problems that will foster pupils to engage in memorizing algorithms rather than understanding mathematical concepts and relations (Schoenfeldt, 2007). Therefore, pupils are more likely to choose unsuccessful superficial imitative reasoning over creative reasoning when solving complex mathematical problems. Furthermore, studies have shown that creative reasoning will increase pupils' abilities to solve mathematical problems (LICR 2009-2012).

Other positive effects in mathematics education have been found within the research field of ICT and learning. ICT may, among other things, support pupils' learning through visualization, simulation and stimulate creativity.

The study, here presented, aims at illuminating under what circumstances ICT can foster pupils to engage in creative reasoning for learning. The study will focus pupils' mathematical reasoning while they, working in pairs, use Geogebra for mathematical problem solving. The mathematical problems concern linear functions, and was chosen in order to challenge the pupils' understanding of the transformation between reality, the algebraic and the graphical representation of functions. The pupils' reasoning will be documented through sound recording, screen recording and recall interviews.

In order to examine the relation between what ICT can offer and pupils' engagement in creative reasoning, data will be analyzed using concepts like reasoning, anchoring, argumentation (Lithner, 2008) and affordances (Greeno, 1994)

References:

Brousseau, G.(1997), Theory of didactical situations in mathematics, Kluwer Academic Publisher

Greeno, JG. (1994), Gibson's affordances. Psychological Review, Vol 101(2), 336-342.

Lithner, J.(2008), A research framework for creative and imitative reasoning. Educ Stud Math 67:255–276

Schoenfeld, A. (2007). Method. In F. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 69–107). Charlotte, NC: Information Age Publishing.

Title: Geogebra as means to rise the effectiveness of a lesson

By: Allar Veelmaa, Loo Secondary School , Estonia

Abstract: The times have passed when the teacher made all the drawings on the blackboard with the chalk and ruler. This has come from the time has decreased and the teacher and the pupil can use different computer programs, what help to make precise drawings quickly, at the same time looking how the drawing is made.

For several years in the Loo school mathematic lessons GeoGebra is used, especially solving the labor-intensive problems (the modification of functions, the extremum problems etc). In the younger classes we use GeoGebra mainly when solving the construction problems (for example relations in the triangle or in the polygon). The pupils more interested in Mathematics can solve complicated geometry problems what they would not do with the help of paper and pencil. Many of the Loo school pupils have participated in the Geogebra thematic competitions organised by the Tiger Leap Foundation.

Many of the Geogebra worksheets used in the lessons can be got by the pupils after the lessons too, because they can be found in the mathematic teachers web addressed <http://www.allarveelmaa.com/>